AA214 – MIDTERM 1999 – October 27, 1999 **DUE: November 1, 1999**
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Take Home, 25 Points

Instructions: Show all your work. Just writing the answer will not get you credit. If you get bogged down with some algebra, write out how you would proceed to complete the problem (it will get you some credit). The points for each problem are given along with hints and helpful information. Make sure you read the problem completely before proceeding.

1. Using the Taylor Table approach on the finite difference approximation of the 1^{st} derivative
$$\left(\frac{\partial u}{\partial x}\right)_j + c\left(\frac{\partial u}{\partial x}\right)_{j-\alpha} = \left(au_j + bu_{j-1}\right)/\Delta x$$
   (a) Find the coefficients a, b, and c in terms of α which minimize the error er_t. (Points:4)
   (HINT: $u_{j-\alpha} = u_j - \alpha\Delta x\left(\frac{\partial u}{\partial x}\right)_j + \frac{1}{2!}(\alpha\Delta x)^2\left(\frac{\partial^2 u}{\partial x^2}\right)_j - \frac{1}{3!}(\alpha\Delta x)^3\left(\frac{\partial^3 u}{\partial x^3}\right)_j + \cdots$*)*
   (b) Find the resulting expression for er_t, in terms of α and find the value of α which *further* minimizes the error. (Points:4)

2. Find the expression for the modified wave number of the scheme in terms of Δx and k. Cast the result in terms of $sin's$ and $cos's$ and where indicated use series expansion to identify the accuracy of the scheme.
   (a) $(\delta_x u)_j = \left(u_{j-2} - 4u_{j-1} + 4u_{j+1} - u_{j+2}\right)/(4\Delta x)$ and identify the accuracy of the scheme. (Points:3)
   (b) $(\delta_{xxxx} u)_j = \left(u_{j-2} - 4u_{j-1} + 6u_j - 4u_{j+1} + u_{j+2}\right)/\Delta x^4$ and identify the accuracy of the scheme. (Points:3)
   (HINT: $\delta_{xxxx}e^{ikj\Delta x} = (k^*)^4 e^{ikj\Delta x}$*, find* $(k^*)^4 = k^4 + O(\Delta x^p)$*, that is, don't try to take the 4th root..)*
   (c) $(\delta_{xx}u)_{j-1} + (\delta_{xx}u)_j + (\delta_{xx}u)_{j+1} = 3\left(u_{j-2} - 2u_j + u_{j+2}\right)/(4\Delta x^2)$ **Don't attempt to determine the accuracy, (too algebraically messy), it's a 4^{th} order accurate method** . (Points:2)
   (HINT: get the expression for $(k^*)^2$*)*

3. Consider the predictor- corrector method
$$\begin{aligned}\tilde{u}_{n+1} &= u_{n-1} + 2h(\tilde{u}')_n\\ u_{n+1} &= u_n + h(\tilde{u}')_{n+1}\end{aligned}$$
applied to the representative equation
$$u' = \lambda u + ae^{\mu t}$$
   (a) Identify the characteristic and particular operators as discussed in class, $[P(E)]$ and $\vec{Q}(E)$ and find the characteristic polynomial $P(E)$. (Points:3)
   (b) Find the σ's for this method *(HINT: it is a 2 root method).* (Points:2)
   (c) Identify the principal and spurious roots and justify your choice. (Points:2)
   (d) Find er_λ and identify the order of this method. (Points:2)
   (e) Find the particular solution,u_∞. (Optional:Points:1)
   (f) Determine the stability of the method, i.e., conditions on λh.(Optional:Points:2)

(**Note***: The* $\sim$ *in* $(\tilde{u}')_n$ *for the predictor step)*